OCR
RECOGNISING ACHIEVEMENT

Friday 17 May 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle is attached to a spring and suspended vertically from a point P which is made to oscillate vertically. The vertical displacement, x, of the particle from a fixed point at time t is modelled by the differential equation

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} + x = \cos t.$$

(i) Find the general solution of the differential equation. **[8]**

Initially the displacement and velocity of the particle are both zero.

(ii) Find the particular solution and sketch its graph for large positive values of t. **[6]**

(iii) Find approximate values of the displacement and velocity at $t = 10\pi$. **[3]**

The point P stops oscillating at $t = 10\pi$ and the subsequent motion of the particle is modelled by

$$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0.$$

(iv) Determine the type of damping present. **[2]**

(v) Using the values obtained in part **(iii)**, find the particular solution for this motion. **[5]**

2 In this question take $g = 10$.

A rocket of mass 500 kg is launched from rest from the sea bed at a depth of 124 m. It travels vertically upwards. After t s it has risen x m and its velocity is $v\,\mathrm{m\,s^{-1}}$.

In a simple model, for all stages of its motion, the mass of the rocket is constant and the only forces acting on it are its weight, a driving force of 10 000 N and a resistance force.

When in the sea, the magnitude of the resistance force is modelled by kv N, where k is a constant.

(i) Write down and solve a differential equation to show that $v = \frac{5000}{k}\left(1 - \mathrm{e}^{-\frac{kt}{500}}\right)$. **[8]**

(ii) Find x in terms of t and k. **[3]**

The time for the rocket to reach the surface of the sea is 5 s.

(iii) Verify that $k \approx 2.5$ is consistent with this information and hence estimate the speed of the rocket when it reaches the surface. **[3]**

After the rocket reaches the surface it travels vertically upwards through the air and the magnitude of the resistance force is now modelled by $0.4v^2$ N.

(iv) Show that $v\frac{\mathrm{d}v}{\mathrm{d}x} = 10 - 0.0008v^2$. **[2]**

(v) Solve this differential equation to find the particular solution for v in terms of x. Sketch a graph of this solution, showing the asymptote. **[8]**

3 **(a)** The differential equation $\frac{dy}{dx} + 2y = \sin 2x$ is to be solved.

(i) Find the complementary function and a particular integral. Hence write down the general solution. **[7]**

(ii) Find the particular solution subject to the condition $y = 2$ when $x = 0$. Sketch the solution curve for $x \geqslant 0$. **[4]**

(b) The differential equation $\frac{dy}{dx} + 2y = e^{-x}$ is to be solved.

(i) Use the integrating factor method to find the general solution for y in terms of x. **[5]**

(ii) Find the particular solution subject to the condition $y = 2$ when $x = 0$. **[2]**

(c) The differential equation $\frac{dy}{dx} + 2y = \tan x$ is to be solved subject to the condition $y = 2$ when $x = 0$.

Use an integrating factor and the approximation $\int_0^1 e^{2x} \tan x \, dx \approx 2.71862$ to calculate an approximate value of y when $x = 1$. **[6]**

4 The simultaneous differential equations

$$\frac{dx}{dt} = x - 2y - z$$
$$\frac{dy}{dt} = x + 3y + z$$
$$\frac{dz}{dt} = -z$$

are to be solved. When $t = 0$, $x = 1$, $y = 0$ and $z = 2$.

(i) Use the third equation to find the particular solution for z in terms of t. **[2]**

(ii) Using part **(i)** eliminate y and z to obtain a second order differential equation for x. Hence find the general solution for x in terms of t. **[12]**

(iii) Find the corresponding general solution for y. **[3]**

(iv) Find the particular solutions for x and y. **[4]**

(v) Show that $x = y$ when $3 \sin t = e^{-3t}$. Deduce that $x = y$ occurs infinitely often. **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

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4758/01 Differential Equations

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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	
	(answer space continued on next page)

1 (i)	**(continued)**
1 (ii)	

1 (iii)	
1 (iv)	
1 (v)	

2 (i)	

2 (ii)	
2(iii)	

2(iv)

2(v)

(answer space continued on next page)

2 (v)	**(continued)**

3 (a) (i)	
	(answer space continued on next page)

3 (a) (i)	**(continued)**
3 (a) (ii)	

3 (b) (i)

3 (b) (ii)

3 (c)

4 (i)

4 (ii)

(answer space continued on next page)

4 (ii)	**(continued)**

4 (iii)

4 (iv)

4 (v)	

OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4758:** Differential Equations

Mark Scheme for June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

Question			Answer	Marks	Guidance	
1	(i)		$2\lambda^2 + 3\lambda + 1 = 0$	M1	Auxiliary equation	
			$\lambda = -1, -\frac{1}{2}$	A1	Correct roots	
			CF $x = A\mathrm{e}^{-t} + B\mathrm{e}^{-t/2}$	F1	FT roots	
			PI $x = a\cos t + b\sin t$	B1		
			$\dot{x} = -a\sin t + b\cos t$, $\ddot{x} = -a\cos t - b\sin t$			
			$2(-a\cos t - b\sin t) + 3(-a\sin t + b\cos t) + (a\cos t + b\sin t) = \cos t$	M1	Differentiate and substitute	
			$\left.\begin{aligned} -a + 3b &= 1 \\ -b - 3a &= 0 \end{aligned}\right\} \Rightarrow a = -\frac{1}{10}, b = \frac{3}{10}$	M1 A1	Compare and solve Correct values	
			GS $x = \frac{1}{10}(3\sin t - \cos t) + A\mathrm{e}^{-t} + B\mathrm{e}^{-t/2}$	F1 **[8]**	FT CF with 2 arbitrary constants + PI	
1	(ii)		$x = 0, t = 0 \Rightarrow 0 = -\frac{1}{10} + A + B$	M1	Use condition	
			$\dot{x} = \frac{1}{10}(3\cos t + \sin t) - A\mathrm{e}^{-t} - \frac{1}{2}B\mathrm{e}^{-t/2}$	M1	Differentiate GS from **(i)**	
			$\dot{x} = 0, t = 0 \Rightarrow 0 = \frac{3}{10} - A - \frac{1}{2}B$	M1	Use condition	
			$x = \frac{1}{10}(3\sin t - \cos t) + \frac{1}{2}\mathrm{e}^{-t} - \frac{2}{5}\mathrm{e}^{-t/2}$	A1	Correct expression	
			Graph	B1	At least one and a half oscillations; scales not required	Note that only large t required, but accept any evidence of oscillations
				B1 **[6]**	Approximately constant amplitude and constant frequency over at least two oscillations; scales not required	FT values of a,b,A,B and both values of λ negative
1	(iii)		$x \approx \frac{1}{10}(3\sin 10\pi - \cos 10\pi) = -\frac{1}{10}$	M1	Use $t = 10\pi$ in either x or $\dot{x}$ soi	
				A1	FT incorrect a and b from (i)	
			$\dot{x} \approx \frac{1}{10}(3\cos 10\pi + \sin 10\pi) = \frac{3}{10}$	A1 **[3]**	FT incorrect a and b from (i)	
1	(iv)		$3^2 - 4 \cdot 2 \cdot 1 = 1$	M1	Consider auxiliary equation; either discriminant or nature of roots found in (i). Or state the CF again.	
			> 0 or state roots are real and distinct. Overdamped	A1 **[2]**	Must give a reason	

Question			Answer	Marks	Guidance	
1	(v)		$x = C\,\mathrm{e}^{-t} + D\,\mathrm{e}^{-t/2}$	B1	FT from (i)	
			$-\frac{1}{10} = C\,\mathrm{e}^{-10\pi} + D\,\mathrm{e}^{-5\pi}$	M1	Use condition. Accept applied at $t = 0$	
			$\dot{x} = -C\,\mathrm{e}^{-t} - \frac{1}{2}D\,\mathrm{e}^{-t/2} \Rightarrow \frac{3}{10} = -C\,\mathrm{e}^{-10\pi} - \frac{1}{2}D\,\mathrm{e}^{-5\pi}$	M1	Use condition. Accept applied at $t = 0$	
			$C = -\frac{1}{2}\mathrm{e}^{10\pi}, D = \frac{2}{5}\mathrm{e}^{5\pi}$	A1	cao $-2.2\times10^{13}\,\mathrm{e}^{-t} + 2.65\times10^{6}\,\mathrm{e}^{-t/2}$	
			$x = -\frac{1}{2}\mathrm{e}^{10\pi - t} + \frac{2}{5}\mathrm{e}^{(10\pi - t)/2}$	A1	FT their C and D if correct t used sc Award for $x = -\frac{1}{2}\mathrm{e}^{-t} + \frac{2}{5}\mathrm{e}^{-t/2}$ if new t clearly defined.	
				[5]		
2	(i)		**EITHER:** $500\frac{\mathrm{d}v}{\mathrm{d}t} = 10\,000 - 500g - kv$	M1	N2L	
			$\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - \frac{k}{500}v$	A1		
			$\int \frac{1}{10 - \frac{k}{500}v}\mathrm{d}v = \int \mathrm{d}t$	M1	Separate and integrate	
			$-\frac{500}{k}\ln\left\lvert 10 - \frac{k}{500}v\right\rvert = t + c$	A1	LHS	
				A1	RHS	
			$10 - \frac{k}{500}v = A\,\mathrm{e}^{-kt/500}$	M1	Rearrange, dealing properly with constant	
			$t = 0, v = 0 \Rightarrow A = 10$	M1	Use condition	
			$v = \frac{5000}{k}\left(1 - \mathrm{e}^{-kt/500}\right)$	E1		
				[8]		
			OR: $500\frac{\mathrm{d}v}{\mathrm{d}t} = 10\,000 - 500g - kv$	M1	N2L	
			$\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - \frac{k}{500}v$	A1		
			IF: $\mathrm{e}^{kt/500}$	B1		
			$v\mathrm{e}^{kt/500} = \int \mathrm{e}^{kt/500}.10\,\mathrm{d}t$	M1	Multiply through by IF and recognise LHS	

Question			Answer	Marks	Guidance	
			$= \frac{5000}{k} e^{kt/500} + A$	A1	Integrate RHS	
			$t = 0, v = 0 \Rightarrow A = -\frac{5000}{k}$	M1	Use condition	
				M1	Rearrange	
			$v = \frac{5000}{k}\left(1 - e^{-kt/500}\right)$	E1		
				[8]		
			OR: $500\frac{dv}{dt} = 10\,000 - 500g - kv$	M1	Use N2L	
			$\frac{dv}{dt} = 10 - \frac{k}{500}v$	A1		
			Auxiliary equation: $\lambda + \frac{k}{500} = 0$	M1		
			CF: $v = Ae^{-\frac{kt}{500}}$	A1		
			PI: $v = \frac{5000}{k}$	M1	Find PI	
			GS: $v = Ae^{-\frac{kt}{500}} + \frac{5000}{k}$	A1		
			$t = 0, v = 0 \Rightarrow A = -\frac{5000}{k}$	M1	Use condition	
			$v = \frac{5000}{k}\left(1 - e^{-kt/500}\right)$	E1		
				[8]		
2	**(ii)**		$x = \int v\,dt = \frac{5000}{k}\left(t + \frac{500}{k}e^{-kt/500}\right) \quad (+c_2)$	M1	Attempt to integrate both terms	
			$t = 0, x = 0 \Rightarrow \frac{5000}{k}\left(\frac{500}{k}\right) + c_2 = 0$	M1	Use condition	
			$x = \frac{5000}{k}\left(t + \frac{500}{k}\left(e^{-kt/500} - 1\right)\right)$	A1	cao	
				[3]		

Question			Answer	Marks	Guidance	
2	(iii)		$k = 2.5, t = 5 \Rightarrow x = \frac{5000}{2.5}\left(5 + \frac{500}{2.5}\left(\mathrm{e}^{-12.5/500} - 1\right)\right)$	M1		
			$= 124.0$ (1dp) i.e. consistent	E1		
			$v = \frac{5000}{2.5}\left(1 - \mathrm{e}^{-12.5/500}\right) \approx 49.4$ to 1dp	B1	cao	
				[3]		
2	(iv)		$500v\frac{\mathrm{d}v}{\mathrm{d}x} = 10\,000 - 500g - 0.4v^2$	M1	N2L	
			$v\frac{\mathrm{d}v}{\mathrm{d}x} = 10 - 0.0008v^2$	E1		
				[2]		
2	(v)		$\int \frac{v}{10 - 0.0008v^2}\mathrm{d}v = \int \mathrm{d}x$	M1	Separate variables: attempt to integrate	
			$-\frac{1}{0.0016}\ln\left\|10 - 0.0008v^2\right\| = x + c_3$	A1 A1	LHS RHS	
			$10 - 0.0008v^2 = B\,\mathrm{e}^{-0.0016x}$	M1	Rearrange, dealing properly with constant	
			$x = 124, v = 49.4 \Rightarrow B = 9.8$	M1	Use correct condition	
			$v = \sqrt{1250\left(10 - 9.8\,\mathrm{e}^{-0.0016x}\right)}$	A1	oe	
			Graph	B1 B1	Increasing and condition shown Asymptote at (awrt) 112	Starting from (124, 49.4) FT
				[8]		
3	(a)	(i)	$\lambda + 2 = 0 \Rightarrow \lambda = -2$	M1	Auxiliary equation or IF : $\frac{\mathrm{d}}{\mathrm{d}x}\left(y\mathrm{e}^{2x}\right) = 0$	
			CF $A\,\mathrm{e}^{-2x}$	A1		
			$y = a\cos 2x + b\sin 2x$	B1		
			$y' = -2a\sin 2x + 2b\cos 2x$			
			$-2a\sin 2x + 2b\cos 2x + 2(a\cos 2x + b\sin 2x) = \sin 2x$	M1	Differentiate and substitute	
			$\left.\begin{aligned} -2a + 2b &= 1 \\ 2b + 2a &= 0 \end{aligned}\right\} \Rightarrow a = -\frac{1}{4}, b = \frac{1}{4}$	M1 A1	Compare and solve cao	
			$y = -\frac{1}{4}\cos 2x + \frac{1}{4}\sin 2x + A\,\mathrm{e}^{-2x}$	F1	CF with one arbitrary constant + PI	
				[7]		

Question			Answer	Marks	Guidance	
3	(a)	(ii)	$2 = -\frac{1}{4} + A$	M1	Use condition	
			$y = -\frac{1}{4}\cos 2x + \frac{1}{4}\sin 2x + \frac{9}{4}e^{-2x}$	A1	cao	
			Graph	B1	Starts from (0, 2) with negative gradient	
				B1	Oscillations with constant amplitude and frequency for large x. Scales not required	Their function must have exponential decay term
				[4]		
3	(b)	(i)	$I = \exp\left(\int 2\,dx\right) = e^{2x}$	B1		
			$e^{2x}\frac{dy}{dx} + 2e^{2x}y = e^{x}$			
			$\frac{d}{dx}\left(e^{2x}y\right) = e^{x}$	M1		
			$e^{2x}y = \int e^{x}\,dx$	M1	Integrate	
			$= e^{x} + c$	A1	All correct	
			$y = e^{-x} + ce^{-2x}$	F1	Divide through by IF	
				[5]		
3	(b)	(ii)	$x = 0, y = 2 \Rightarrow 2 = 1 + c \Rightarrow c = 1$	M1	Use condition	
			$y = e^{-x} + e^{-2x}$	A1	cao	
				[2]		
3	(c)		$I = e^{2x}$	M1		
			$\frac{d}{dx}\left(e^{2x}y\right) = e^{2x}\tan x$	M1		
			$\left[e^{2x}y\right]_{x=0}^{x=1} = \int_0^1 e^{2x}\tan x\,dx$	A1	LHS with limits	
				A1	RHS	
			$y(1)e^{2} - 2e^{0} \approx 2.71862$	B1	LHS	
			$y(1) \approx 0.6386$	A1	0.639 or better	
				[6]		
4	(i)		$z = Ae^{-t}$	B1		
			Conditions $\Rightarrow z = 2e^{-t}$	B1		
				[2]		

Question			Answer	Marks	Guidance	
4	(ii)		$\dot{x} = x - 2y - 2\mathrm{e}^{-t}$			
			$\dot{y} = x + 3y + 2\mathrm{e}^{-t}$	M1	Substitute	
			$y = \frac{1}{2}\left(x - \dot{x} - 2\mathrm{e}^{-t}\right)$	M1	Rearrange	
			$\dot{y} = \frac{1}{2}\left(\dot{x} - \ddot{x} + 2\mathrm{e}^{-t}\right)$	M1	Differentiate	
			$\frac{1}{2}\left(\dot{x} - \ddot{x} + 2\mathrm{e}^{-t}\right) = x + \frac{3}{2}\left(x - \dot{x} - 2\mathrm{e}^{-t}\right) + 2\mathrm{e}^{-t}$	M1	Substitute	
			$\ddot{x} - 4\dot{x} + 5x = 4\mathrm{e}^{-t}$	A1	cao	
			$\lambda^2 - 4\lambda + 5 = 0$	M1	Auxiliary equation	
			$\lambda = 2 \pm \mathrm{j}$	A1	Correct roots	
			CF $\mathrm{e}^{2t}\left(B\cos t + C\sin t\right)$	M1	Correct form of CF for their roots	
				F1	FT their roots	
			PI $x = a\mathrm{e}^{-t}$	B1	Correct form of PI	FT if exponential term on RHS of d.e.
			$\dot{x} = -a\mathrm{e}^{-t}$, $\ddot{x} = a\mathrm{e}^{-t}$ $\quad a\mathrm{e}^{-t} + 4a\mathrm{e}^{-t} + 5a\mathrm{e}^{-t} = 4\mathrm{e}^{-t} \Rightarrow a = \frac{2}{5}$	M1	A valid method to find a	
			GS $x = \frac{2}{5}\mathrm{e}^{-t} + \mathrm{e}^{2t}\left(B\cos t + C\sin t\right)$	A1	cao	
				[12]		
4	(iii)		$y = \frac{1}{2}\left(x - \dot{x} - 2\mathrm{e}^{-t}\right)$	M1	Substitute x and $\dot{x}$ into expression for y	
			$\dot{x} = -\frac{2}{5}\mathrm{e}^{-t} + 2\mathrm{e}^{2t}\left(B\cos t + C\sin t\right) + \mathrm{e}^{2t}\left(-B\sin t + C\cos t\right)$	M1	Differentiate using product rule	
			$y = -\frac{3}{5}\mathrm{e}^{-t} + \frac{1}{2}\mathrm{e}^{2t}\left(-(B+C)\cos t + (B-C)\sin t\right)$	A1	cao aef, simplified to a maximum of 5 terms	
				[3]		
4	(iv)		$x = 1, t = 0 \Rightarrow \frac{2}{5} + B = 1$	M1	Use condition	
			$y = 0, t = 0 \Rightarrow -\frac{3}{5} - \frac{1}{2}B - \frac{1}{2}C = 0$	M1	Use condition	
			$B = \frac{3}{5}, C = -\frac{9}{5}$			
			$x = \frac{2}{5}\mathrm{e}^{-t} + \frac{3}{5}\mathrm{e}^{2t}\left(\cos t - 3\sin t\right)$	A1		
			$y = -\frac{3}{5}\mathrm{e}^{-t} + \frac{3}{5}\mathrm{e}^{2t}\left(\cos t + 2\sin t\right)$	A1		
				[4]		

Question			Answer	Marks	Guidance	
4	(v)		$x = y \Leftrightarrow \frac{2}{5}e^{-t} + \frac{3}{5}e^{2t}(\cos t - 3\sin t)$ $= -\frac{3}{5}e^{-t} + \frac{3}{5}e^{2t}(\cos t + 2\sin t)$ $\Leftrightarrow e^{-t} = e^{2t}(3\sin t) \Leftrightarrow e^{-3t} = 3\sin t$	B1	Be convinced	
			For $t > 0$, $0 < e^{-3t} < 1$ so its graph will meet the graph of $3\sin t$ infinitely often	M1 E1 **[3]**	Consider graphs oe Complete argument	

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Unit level raw mark and UMS grade boundaries June 2013 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0
4798/01	(FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241/01	(Z1) Statistics 1	Raw	72	55	48	41	35	29	0
		UMS	100	80	70	60	50	40	0
G242/01	(Z2) Statistics 2	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243/01	(Z3) Statistics 3	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0